MATH 1A - FINAL EXAM

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Name: ____________________

Instructions: This is it, people! Your final hurdle to freedom :) This exam counts for 30% of your grade and you officially have 110 minutes to take this exam (although I will try to give you more time). Please $\boxed{\text{box}}$ your answers.

By the way, enjoy the rest of your $\sum$mer :)

Note: This is the final exam, **NOT** the final exam deluxe. Please sign here to acknowledge this fact: _________________________.

1		20
2		10
3		40
4		20
5		20
6		20
7		10
8		10
Bonus 1		5
Bonus 2		5
Bonus 3		5
Total		150

Date: Friday, August 12th, 2011.

1. *(20 points)* Use the **definition** of the integral to evaluate:

$$\int_0^1 \left(x^3 - 2\right) dx$$

You may use the following formulas:

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Note: -2 for not writing $\lim_{n\to\infty}$

(This page is left blank in case you need more space to work on problem 1)

2. *(10 points)* Evaluate the following limit:

$$\lim_{n\to\infty} \frac{1}{n}\left(e^{\frac{1}{n}} + e^{\frac{2}{n}} + \cdots + e^{\frac{n}{n}}\right)$$

3. *(40 points, 5 points each)* Find the following integrals:

(a) $\int_{-1}^{1} \sqrt{1-x^2}dx$

Note: Don't spend too much time on this one, either you know it or you don't!

(b) $\int \frac{1}{x^2+1}dx$

Note: Ditto!

(c) The antiderivative F of $f(x) = 3e^x + 4\sec^2(x)$ which satisfies $F(0) = 1$.

(d) $\int_0^1 x^3 + x^4 dx$

(e) $g'(x)$, where $g(x) = \int_{x^2}^{e^x} \sin(t^3)dt$

(f) $\int e^x \sqrt{e^x - 1}dx$

(g) $\int_{e}^{e^2} \left(\frac{(\ln(x))^3}{x}\right) dx$

(h) The average value of $f(x) = \sin(x^5)(1 + e^{-x^2} + x^2)$ on $[-\pi, \pi]$

4. *(20 points)* Find the area of the region enclosed by the curves:

$$y = \cos(x) \quad \text{and} \quad y = -\cos(x) \qquad \text{from } 0 \text{ to } \pi$$

Hint: It might help to notice a certain symmetry in your picture!

(This page is left blank in case you need more space to work on question 4.)

5. *(20 points, 10 points each)* Find the following limits

(a) $\lim_{x\to-\infty} \frac{\sqrt{x^2+4}}{x}$

(b) $\lim_{x\to 0^+} x^{x^2}$

6. *(20 points, 10 points each)* Find the derivatives of the following functions

   (a) $f(x) = (\sin(x))^x$

(b) y', where $x^y = y^x$

Hint: Take $\ln$s first, and then differentiate.

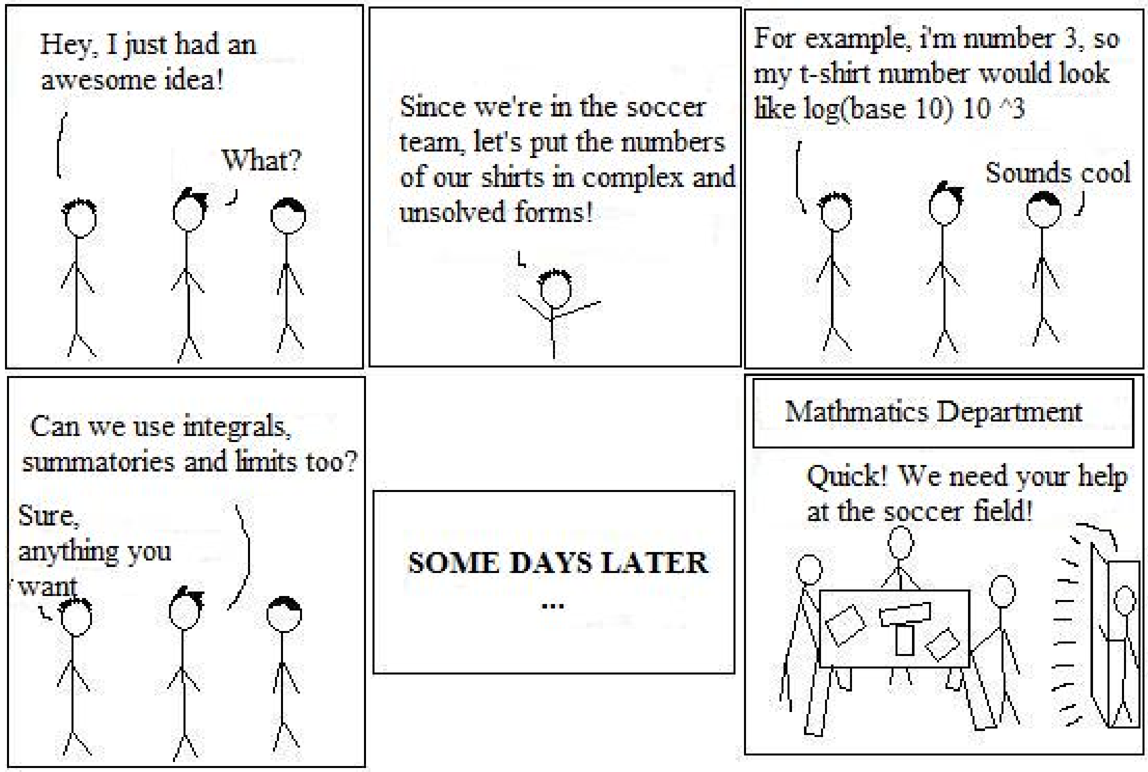
7. *(10 points)* Find the absolute maximum and minimum of the following function on $[0, \frac{\pi}{2}]$:

$$f(x) = \sin(x) + \cos(x)$$

Hint: $\cos(x) = \sin(x)$ when $x = \frac{\pi}{4}$

8. *(10 points)* Who's your favorite Math 1A teacher of all time???

1A/Practice Exams/Soccer.jpg



Bonus 1 *(5 points)* Fill in the gaps in the following proof that the function f is not integrable on $[0,1]$:

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$$

Step 1: Pick x_i^* such that ________________________. Then:

$\int_0^1 f(x)dx =$

Step 2: Pick x_i^* such that ________________________. Then:

$\int_0^1 f(x)dx =$

Since we get two different answers for the integral, we have a contradiction. $\Rightarrow\Leftarrow$. And hence f is not integrable on $[0,1]$.

Note: See the handout 'Integration sucks!!!' for a nice discussion of this problem!

Bonus 2 *(5 points)* Another way to define $\ln(x)$ is:

$$\ln(x) = \int_1^x \frac{1}{t} dt$$

Show **using this definition only** that $\ln(e^x) = x$.

Hint: Let $g(x) = \ln(e^x) = \int_1^{e^x} \frac{1}{t} dt$.

First differentiate g, then simplify, and then antidifferentiate your answer. Make sure you face the issue of the constant!

Bonus 3 *(5 points)* Define the **Product integral** $\prod_a^b f(x)dx$ as follows:

If we define Δx, x_i, and x_i^* as usual, then:

$$\prod_a^b f(x)dx = \lim_{n\to\infty} \left(f(x_1^*)\right)^{\Delta x} \left(f(x_2^*)\right)^{\Delta x} \cdots \left(f(x_n^*)\right)^{\Delta x}$$

That is, instead of summing up the $f(x_i^*)$, we *multiply* them!

Question: Express $\prod_a^b f(x)dx$ in terms of $\int_a^b f(x)dx$

Hint: How do you turn a product into a sum?

Note: In other words, although this *looks* like a new concept, it really isn't, which is quite surprising!

(Scrap work)

Any comments about this exam? (too long? too hard?)

CONGRATULATIONS!!!

You're officially done with this course! :) Thank you so much for having me, and I hope you had a lot of fun! :)

Any other comments or goodbye words?